## To the Editor:

A short calculation method for belt-filterlike multizone countercurrent filtration washing presented by Tosun in 1996 was compared via a numerical example with a long calculation method of Tomiak (1984). Discrepancy was found between the results obtained. Dr. Tosun took his results to be correct and looked for fault in the long method.

Such approach is based on the belief that the two methods are contradictory. This is not the case, and the two methods can be reconciled as shown below.

There are no faults in either treatment. The short method is a special case of the long method, in which the effect of nonuniform top-to-bottom liquor concentration in the filter cake is neglected.

In the short method, binary mixing formula $c_k = h_k c_{k-1} + d_k w_{k+1}$ is applied to all the zones. This corresponds to the same individual zone losses, to which the $h_k = [h_1(N)]^k$ formula applies.

In the long method, the binary mixing formula is applied only to the first zone, where there is uniform top-to-bottom liquor concentration in the filter cake at the start. For the following zones, the formula is extended to multicomponent mixing formulas, by adding one more component per zone to take into account each wash liquor addition. This allows for nonuniform top-to-bottom liquor concentration in filter cake, in which a concentration profile develops as a result of washing. Its mathematical treatment is complex and involves relationships from residence time distribution studies applied to miscible displacement of resident liquor from porous beds by invading wash liquor, which was explained for belt filters in a review paper (Tomiak, 1982).

To find out whether or not the changes in filter cake concentration profile are allowed for, one should compare the displacement ratios (Perkins et al., 1954) (or their complementary fractional holdup values) for individual zones. If they were the same, no allowance for it was made. For the numerical example considered, we have (with $c_o = 1$ for convenience) the following displacement ratios:

| ZONE | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Displacement ratio | $\frac{1-c_1}{1-w_2}$ | $\frac{c_1-c_2}{c_1-w_3}$ | $\frac{c_2-c_3}{c_2-w_4}$ | $\frac{c_3-c_4}{c_3-w_5}$ | $\frac{c_4-c_5}{c_4}$ |
| Short method | 0.7673 | 0.7673 | 0.7673 | 0.7673 | 0.7673 |
| Long method | 0.7673 | 0.7856 | 0.7868 | 0.7869 | 0.7869 |

confirming that no allowance was made in the short method for filter-cake liquor concentration profile, which made the 3.25% loss higher than the 2.82% loss with allowance for the concentration profile.

The long method is quite general and applies to any given simple filtration washing (SFW) curve $f_o^*$ vs. $N$, experimental or based on specific theoretical washing model, defining washing performance of particular filter cakes (Tomiak, 1979). For the calculations, knowledge of the SFW curve at $N$, $2N$, $3N$, ... $nN$ for $n$ washing zones is needed to describe $n$ passes of the wash liquor. This is hard to obtain experimentally at high wash ratios because of small values of the SFW losses $f_o^*(N)$.

Carrying the long method calculations for the SFW losses corresponding to the short method fractional liquor holdups $h_k = (h_1)^k$ [i.e., with $f_o^*(N) = h_1$, $f_o^*(2N) = (h_1)^2$, $f_o^*(3N) = (h_1)^3$, $f_o^*(4N) = (h_1)^4$ and $f_o^*(5N) + (h_1)^5$ in

**Table 1. Long Method Calculations for $h_k = [h_1(N)]^k$ with $h_1(N) = 0.23267$ vs. Results for $h_k = h_1(kN) = (1+kN)e^{-2kN}$ for PMCS Model with Two Cells**

| | | |
|---|---|---|
| $N = 1.1$ | | |
| $h_1 = 2.1e^{-2.2} = 0.23267$ | | |
| $h_2 = (h_1)^2 = 0.05414$ | vs. 0.0393 | for $f_o^* = (1+N)e^{-2N}$ SFW curve |
| $h_3 = (h_1)^3 = 0.01260$ | vs. 0.0059 | |
| $h_4 = (h_1)^4 = 0.00293$ | vs. 0.0008 | |
| $h_5 = (h_1)^5 = 0.00068$ | vs. 0.0001 | |
| $d_2 = h_1 - h_2 = 0.17854$ | | |
| $d_3 = h_2 - h_3 = 0.04154$ | | |
| $d_4 = h_3 - h_4 = 0.00967$ | | |
| $d_5 = h_4 - h_5 = 0.00225$ | | |
| $a = N - 1 + h_1 = 0.33267$ | | |
| $D_2 = d_2/a = 0.53667$ | | |
| $D_3 = (d_3 + d_2 D_2)/a = 0.41289$ | | |
| $D_4 = (d_4 + d_3 D_2 + d_2 D_3)/a = 0.31766$ | | |
| $D_5 = (d_5 + d_4 D_2 + d_3 D_3 + d_2 D_4)/a = 0.24440$ | | |
| $S_5 = 1 + D_2 + D_3 + D_4 + D_5 = 2.51162$ | | |
| $c_5 = [a - S_5(N-1)]/S_5 = 0.03246$ | | vs. $c_5 = 0.0282$ |
| $w_1 = (1 - c_5)/N = 0.87958$ | | vs. $w_1 = 0.8834$ |
| $w_2 = (h_1 - c_5)/a = 0.60185$ | | vs. $w_2 = 0.6145$ |
| $w_3 = (h_2 - c_5 + d_2 w_2)/a = 0.38818$ | | vs. $w_3 = 0.3904$ |
| $w_4 = (h_3 - c_5 + d_3 w_2 + d_2 w_3)/a = 0.22378$ | | vs. $w_4 = 0.2214$ |
| $w_5 = (h_4 - c_5 + d_4 w_2 + d_3 w_3 + d_2 w_4)/a = 0.09731$ | | vs. $w_5 = 0.0948$ |
| $c_1 = c_5 + N w_2 = 0.069450$ | | vs. $c_1 = 0.7042$ |
| $c_2 = c_5 + N w_3 = 0.45945$ | | vs. $c_2 = 0.4577$ |
| $c_3 = c_5 + N w_4 = 0.27862$ | | vs. $c_3 = 0.2718$ |
| $c_4 = c_2 + N w_5 = 0.13950$ | | vs. $c_4 = 0.1325$ |

The fractional washing losses are given by the $c_5$ concentrations and show 3.25 and 2.82% loss for the short and long method.
The above calculations follow Table 1, Appendix 4 in Tomiak (1984).

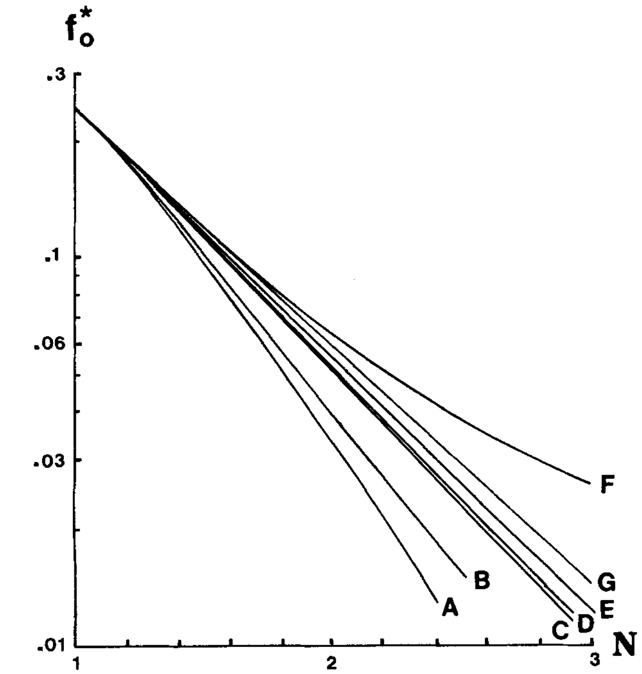


**Figure 1. SWF fitted to ($f_o^* = 0.2455$, $N = 1$) point.**

From Tomiak (1994), with SFW curve for equal zone efficiencies (EZE) equation added.

the example] yields the same results as the short method calculations of Tosun (1996) (see Table 1).

The involvement of the PMCS model in the short method is limited to locating the ($f_o^* = h_1$, $N$) point on the SFW curve. Other models may be used for this purpose or one can start with an experimental ($h_1$, $N$) point. The short method may thus be used for rough extrapolation of given $f_o^*(N) = h_1(N)$ to higher wash ratios via the $h_k = [h_1(N)]^k$ formula. The calculations refer to the SFW curve starting at the given SFW loss at wash ratio $N$, but then deviating from the generally unknown actual SFW curve. The SFW curve for the case is represented on semilog paper by a straight line passing through ($f_o^* = 1$, $N = 0$) and given ($f_o^*$, $N$) point. Such linear plots are reasonable as illustrated in Figure 1, which shows SFW curves based on Table 2 equations fitted to ($f_o^* = 0.2455$, $N = 1$) point.

The straight line relationship was originally proposed by Choudhury and Dahlstrom (1957) via $f_o^* = [1 - (E'/100)]^N$ equation (with wash efficiency $E'$ defined as % solute removed from filter cake at wash ratio $N = 1$). Wash efficiency values have been found to vary from a minimum of 35 to a maximum of 86, compared with $E' = 100(1 - e^{-1}) = 63.2$ value for the PMCS model with single cell ($j = 1$, for which $f_o^* = e^{-N}$). It should be noted that for this special case, the same results are obtained for the short and long method, because by virtue of complete mixing there is no liquor concentration profile and the individual zone efficiencies are the same and we have $f_o^*(kN) = [f_o^*(N)]^k$.

The PMCS model allows obtaining mathematical solutions without residence time distribution considerations, because it in effect converts nonequilibrium stages to equilibrium stages. This makes mathematics tractable in straightforward, albeit complicated way.

**Table 2. Theoretical Equations for SFW Curves**

| | |
|---|---|
| Dispersion Model[a] with $Pe = uL/D_L$ ($Pe = 4$ typical) | $w^o = 1 - 1/2\left\{\mathrm{erfc}\left[\frac{1-N}{2\sqrt{N}}\sqrt{Pe}\right] + \exp(Pe)\cdot\mathrm{erfc}\left[\frac{1+N}{2\sqrt{N}}\sqrt{Pe}\right]\right\}$ <br> $f_o^* = 1 - \int_0^N w^o\, dN$ |
| Diffusion model [b] with $\bar{P} = uL/4D$ ($\bar{P} = 0.8$ typical) | $f_o^* = f(\bar{P}, N) = \bar{c}$ with numerical values of $f_o^*$ available from tables for various $N$s for constant values of $\bar{P}$. |
| Perfect mixing cells in series (PMCS)[c] model with $j$ cells ($j = 2$ typical) | $f_o^* = \frac{1}{j}[j + (j-1)\frac{jN}{1!} + (j-2)\frac{(jN)^2}{2!} + \ldots + \frac{(jN)^{(j-1)}}{(j-1)!}]e^{-jN}$ <br> For $j = 2$ we have $f_o^* = (1+N)e^{-2N}$ <br> For $j = 1$ we have $f_o^* = e^{-N}$ |
| Mass transfer model [d],[e] ($N_{OY} = 4$ typical) | $f_o^* = f(N_{OY}, N)$ with numerical values of $f_o^*$ available from tables for various $N$s for constant values of mass transfer units $N_{OY}$. |
| Plug flow/Perfect mixing model [f] ($x = 0.3$ typical) | $f_o^* = (1-x)e^{-((N-x)/(1-x))}$ for $N \geq x$ <br> $f_o^* = 1 - N$ for $N \leq x$ |
| Norden's efficiency factor equation ($E = 3$ typical) | $f_o^* = \frac{N-1}{N^{E+1}-1}$ for $N \neq 1$ <br> $f_o^* = \frac{1}{E+1}$ for $N = 1$ |
| Equal zone efficiency (EZE) equation ($y = 0.25$ typical) | $f_o^* = y^N$ with $y = f_o^*(N = 1)$. |

From Tomiak (1994) with EZE equation added.
(a) Wakeman (1980) (d) Norden and Viljakainen (1980)
(b) Brenner (1962) (e) Norden et al. (1982)
(c) Tomiak (1973) (f) Moncrieff (1965)

It involves series of calculations based on algebraical formulas given for five countercurrent zones for $j = 5$ (Tomiak, 1973) and $j = 2$ (Tomiak and Lauzon, 1978). Two rounds of calculations are needed: the first for an assumed high value of washings concentration $w_1$ and the second to match given wash liquor concentration $w_o$ ($w_o = 0$ for solute-free wash liquor) via nonsolute-free wash liquor transformations explained in the *Pulp Washing Calculation Manual* (Tomiak, 1994). The algebraical formulas were derived in another, more general way (Tomiak, 1974). The calculations yield the same results as the long method, which is simpler.

The long method was explained in different ways via intuitive reasoning (Tomiak, 1979), the superposition principle (Norden et al., 1982), and multicomponent mixing (Tomiak, 1984) considerations.

In scrutinizing different treatments of filtration washing calculations, the pertinent question is: What SFW curve it corresponds to? For the short method, the answer is: The SFW curve assuming the same losses at all zones, which (except for the case of perfect mixing of liquor in filter cake for which the treatment is exact), is an approximation neglecting the filter cake liquor concentration profile. For the long method, the answer is: It corresponds to any SFW curve, i.e., the method predicts multizone filtration washing results for known SFW behavior defined by specific mathematical models or by given experimental points at $N$, $2N$, ... $nN$ wash ratios.

## Summary

The following calculation methods are available for the belt-filterlike multizone countercurrent filtration washing:

1. The "long" method called the SFW curve method provides exact solutions for given SFW curves.
2. The "short" method called the equal zone efficiency (EZE) method provides approximate solutions based on rough extrapolation of given SFW loss $h_1$ at wash ratio $N$.
3. The PMCS model method, which may be used as a check of the SFW curve method, provides exact solutions for the SFW curves for the PMCS model and was proposed before the SFW curve method was developed.

Because of diminishing contributions of higher wash ratio terms, the equal zone efficiency method provides reasonable approximations in the range of wash ratios of practical interest. With the uncertainty of choosing the right washing model, the equal zone efficiency method has a potential use as a standard case for computer simulation studies.

The explanations provided here are needed to avoid uncritical acceptance of shortcut calculation methods, which at a first glance appear plausible but on closer scrutiny are approximations requiring careful interpretation and review of hidden assumptions.

*Note:* The word zone in this letter was used for what was called stage in some references. This follows pulp and paper terminology, where the term washing stage denotes part of a washing system centering around filter and including auxiliary equipment and the term zone denotes a part of washing section of the filter over which wash liquor at the same concentration is distributed and variable concentration washings are drained from the filter cake. From theoretical point of view, the zones discussed here are nonequilibrium stages referred to in titles of Tomiak (1984) and Tosun (1996), and the subject matter is of wider interest than numerical comparison of two calculation methods.

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